**Proceedings of the**
**International Conference on Mechanical Engineering 2011**
**(ICME2011) 18-20 December 2011, Dhaka, Bangladesh**

**ICME11-FL-049**

# NUMERICAL STUDY FOR FORCED CONVECTIVE TURBULENT FLOW IN A RECTANGULAR ELBOW

**Snehamoy Majumder , Debasish Roy , Rabin Debnath, Arindam Mandal**
**and Somnath Bhattacharjee**

Department of Mechanical Engineering, Jadavpur University, INDIA,

## ABSTRACT

In this paper, the forced convective heat transfer of turbulent fluid flow is studied numerically in a rectangular elbow at a prescribed uniform wall temperature and specified inlet conditions. The analysis of the turbulent flow has been carried out in two dimensional Cartesian co-ordinates considering steady, incompressible and non-reacting fluid flow through the elbow for the sake of simplicity. The working fluid is air and it is admitted through the inlet of the duct at a temperature less than the wall temperature. The velocity, temperature and local average Nusselt number distributions in the whole flow field are estimated numerically by solving the Navier–Stokes and the Energy equations along-with standard $\kappa-\varepsilon$ turbulent model using FLUENT 6.3.26.

**Keywords:** Rectangular Elbow, Turbulent Flow, $\kappa-\varepsilon$ Equation

## 1. INTRODUCTION

The turbulent forced heat transfer in ducts is encountered in different Industrial applications. Laminar fluid flow with convective heat transfer through rectangular duct is usually encountered in a good number of engineering applications such as solar collectors, concentrators, heat exchangers and cooling of modern electronic equipments etc. The analysis becomes complicated when the flow become turbulent. The analysis of such a flow in a rectangular elbow is important in connection with several Engineering applications like heat exchanger, fluid transport ducting system, air conditioning devices etc. The analysis of such flow is complicated owing to the complicacy of the geometry. Apart from this, the presence of non-uniformity of the flow enhances the severity of the problem concerned to be analyzed.

Review of literatures reveals that many attempts have been made to numerically simulate forced convective turbulent flow in the horizontal and vertical straight ducts of constant cross section. Cheng et al. [1] studied numerically combined free and forced convection heat transfer through horizontal rectangular channel under buoyancy conditions of axially uniform heat flux (UHF) and peripherally uniform wall temperature (UWT) for steady fully developed flow by using the methods of Patankar and Spalding [2]. Cheng and Weng [3] carried out numerical study on vertical rectangular duct with one wall uniform high temperature for developing flow of mixed convection. Payvar [4] demonstrated his numerical results for heat transfer enhancement in laminar flow of viscoelastic fluids through rectangular ducts. Yang and Kuan [5] obtained the mean and turbulent flow velocities of gas and particulate phases inside a curved $90^0$ bend. Gas-liquid two phase turbulent flow simulation for a $90^0$ bend elbow was reported by Spedding and Benard [6]. An experimental investigation of turbulent fluid flow through a rectangular elbow with estimation of coefficient of friction was made by Mandal *et al* [7]. A comparative numerical study between three types of turbulent models e.g. standard k-ε, k-ω and Reynolds Stress models with flow visualization for only the turbulent air flow in a two-dimensional rectangular elbow has been subsequently made by the same authors [8]. Özisik [9] derived some expressions for friction factor, pressure drop and heat transfer coefficient for the turbulent forced convection flow in circular ducts only. The turbulent flow and forced convective heat transfer in both straight and wavy ducts, with rectangular, trapezoidal and triangular cross-sections, under fully developed conditions, were numerically predicted by Rokni and Gatski [10]. Majumder and Sanyal [11] studied the relaminarization of Turbulent fluid flow numerically. The developing mixed convection heat transfer in inclined rectangular ducts with wall transpiration effects had been numerically investigated by Jang et al. [12].

In the present paper, the turbulent fluid flow with forced convective heat transfer is studied numerically in a two-dimensional rectangular elbow having a prescribed uniform wall temperature (UWT) and different inlet conditions.

## 2. GEOMETRICAL DESCRIPTION

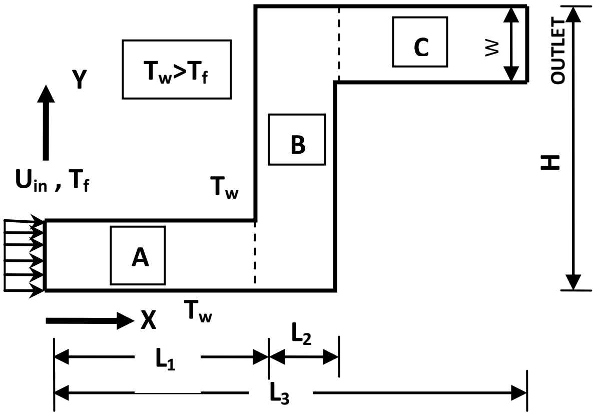

Fig 1. Schematic Diagram of the two dimensional rectangular elbow

The geometry is shown in figure 1 and the dimensions are $L_1$=0.10 m, $L_2$=0.05 m, $L_3$=0.25 m, height, H=0.20 m and width W=0.05 m. The three part lengths of the elbow are considered sufficient to ensure thermally fully developed conditions at the duct out let.

## 3. MATHEMATICAL MODELING

The Navier–Stokes equations and the Energy equation in two-dimensional Cartesian co-ordinate system are solved to determine the velocity and temperature fields. The standard $\kappa-\varepsilon$ turbulent model equation along with the standard model constants is also considered to be used for the steady, incompressible and non-reacting fluid having constant density ρ, viscosity μ in a two-dimensional rectangular elbow. These equations are as follows:

### 3.1.1. Navier–Stokes Equations

*Continuity Equation*

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \qquad (1)$$

*Momentum Equation*

X- Component:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

Y- Component:

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

### 3.1.2. Energy Equation

The temperature distribution of the fluid is described by the energy equation in two-dimensional Cartesian co-ordinate system.

$$\rho C_p u\frac{\partial T}{\partial x}=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (4)$$

### 3.1.3. $\kappa-\varepsilon$ turbulent model Equation

*k Equation*

$$\frac{\partial k}{\partial t}+\bar{u}_k\frac{\partial k}{\partial x_k}=\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_k}+\frac{\partial \bar{u}_k}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_k}+\frac{\partial}{\partial x_k}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_k}\right)-\varepsilon+\nu\nabla^2 k \qquad (5)$$

*ε Equation*

$$\frac{\partial \varepsilon}{\partial t}+\bar{u}_i\frac{\partial \varepsilon}{\partial x_i}=\nu\nabla^2\varepsilon+\frac{\partial}{\partial x_k}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_k}\right)+c_1\frac{\varepsilon}{k}\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_k}+\right.$$
$$\partial uk\partial xi\partial ui\partial xk{-}c2\varepsilon 2k$$

(6)

Here, $C_1, C_2, \sigma_k$ and $\sigma_\varepsilon$ are the empirical turbulence constants, and some typical values of these constants in the standard $\kappa-\varepsilon$ model are recommended by Launder and Spalding [13] which are given below-

$C_1$ = 1.44, $\sigma_k$ = 1.0, $C_2$ = 1.92, $\sigma_\varepsilon$ = 1.3

$$G=\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_k}+\frac{\partial \bar{u}_k}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_k} \qquad (7)$$

### 3.1.4. Nusselt number

$$N_u=\frac{Qconvective}{Qcondutive}=\frac{h\frac{dT}{dy}.A}{k.(T_b-T_i)}=\frac{h}{k}\cdot\frac{1}{(T_b-T_i)}\cdot\frac{dT}{dy} \qquad (8)$$

In this case, the convective Nusselt number $N_{u(conv)}$ can be computed from the definition of the local Nusselt number,

$$N_u=\frac{q.D_h}{k(T_w-T_b)} \qquad (9)$$

Where q is the convective heat flux [W/m$^2$] depending on the quantity to be evaluated. The bulk mean temperature $T_b$ at the duct cross-section is given by,

$$T_b\,(X) = \frac{1}{u(X)\,A}\int u(x,y,z)\,T(x,y,z)\,dA \qquad (10)$$

Where u(X) is the average fluid velocity at any cross-section of the duct.
The average Nusselt number for each cross-section along the heated wall is then evaluated from

$$N_{u(avg)} = \frac{\sum_{Y=0}^{Y=1} N_u}{N} \qquad (11)$$

Where N is the number of Nu nodal values on the heated wall.

### 3.2. Boundary Conditions

All the equations given in articles 3.1.1 to 3.1.3 are solved for the following cases of the different boundary conditions:
(i) A steady fully developed turbulent of a viscous incompressible Newtonian air flow is admitted in an elbow duct of rectangular cross-section.

(ii) u (0,y) = u(y,0)= 0; $u_{inlet}$ = 5 m/s

(iii) $T_{inlet} = T_f$ =300K; $T_{Wall} > T_f$ ; $T_{wall}$ = 330K

(iv) Outflow is fully developed; No-slip at walls

### 3.3. Fluid Properties

The fluid is supplied through the duct at a prescribed uniform speed 5m/s and at a temperature 300K. The fluid used is air of density $\rho$ = 1.225kg/m$^3$ and viscosity $\mu$ = 1.7894X10$^{-5}$ kg/(m.s). The duct material is considered Aluminium metal of thickness 0.02 m, density $\rho$ = 2719kg/m$^3$ and thermal conductivity k = 202.4W/m-K.

## 4. RESULTS AND DISCUSSION

In the Fig 2 & 3, it has been shown that a simple uniform air flow through the inlet of an elbow is producing several re-circulation bubbles residing at different zones of the elbow. From the schematic views of velocity vector and streamlines contour diagrams; it is obvious that near the corners, there are strong recirculation flows which change the configuration of the main flow field

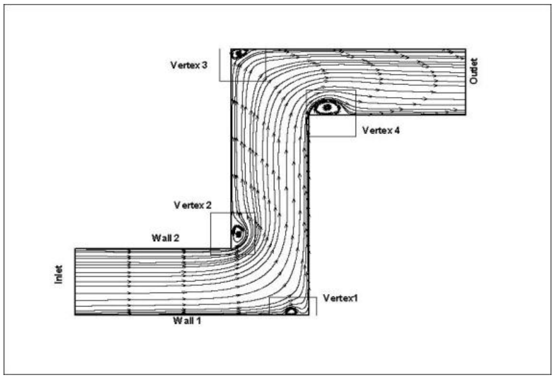

Fig 2, Streamline Contour

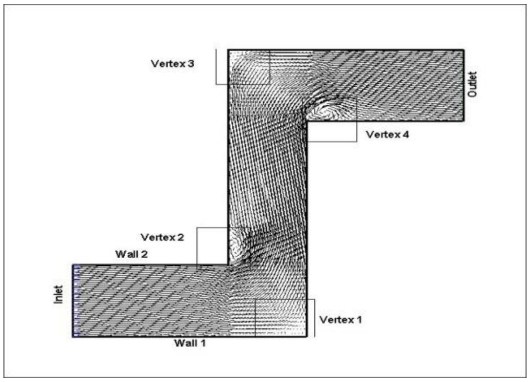

Fig 3, Velocity Vector

Here, in the lower part of the upper horizontal limb there exists a large recirculation bubble along with recirculation bubbles at corner zones.

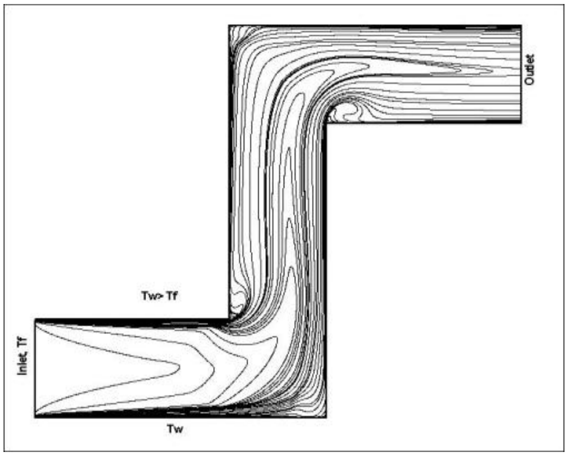

Fig 4, Temperature Contour

Fig 4 and 5 show the temperature contour in between the temperature range 300-330K.

In the Fig 6, 7 and 8, velocity profiles at different positions corresponding to three parts of the elbow

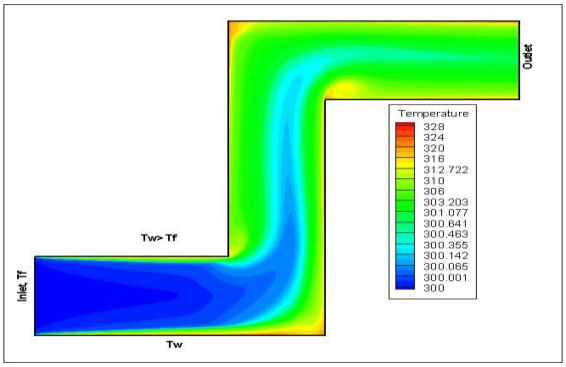

Fig 5, Flooded temperature Contour

have been shown. Fig 6 shows no re-circulation in the lower horizontal limb, whereas re-circulations are prominent at the positions Y=0.07m and Y=0.19m respectively at the vertical limb and similar phenomenon occurs at position X=0.17m of the upper horizontal limb.

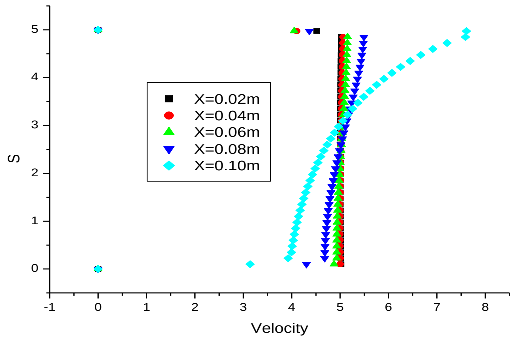

Fig 6. Velocity distribution at different X (A)

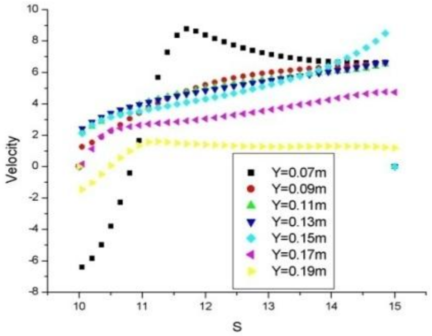

Fig 7. Velocity distribution at different Y (B)

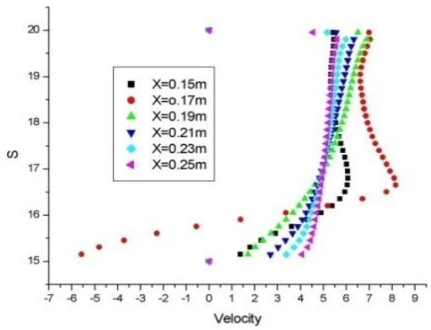

Fig 8, Velocity distribution at different X(C)

In the Fig 9, 10 and 11 temperature variations at different heights and distances have been illustrated. In the lower limb at X=0.1m temperature is almost constant irrespective of the height whereas it increases at other distances as shown in the Fig 10.

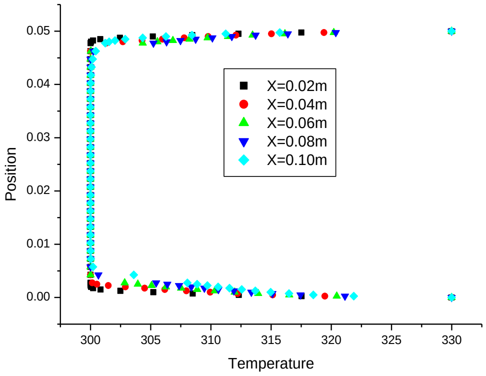

Fig 9. Temperature distribution at different X(A).

At the middle limb temperature remains almost constant at various horizontal distances as shown in the Fig 10. In the upper limb in Fig 11, temperature is constant along the horizontal axis and it increases almost asymptotically.

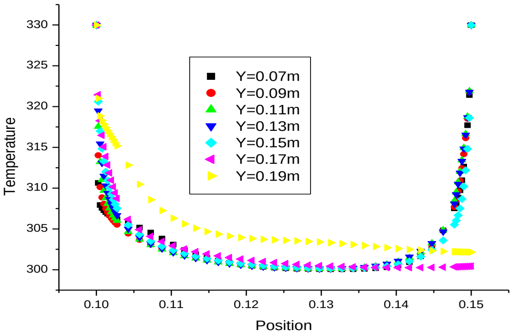

Fig 10. Temperature distribution at different Y(B)

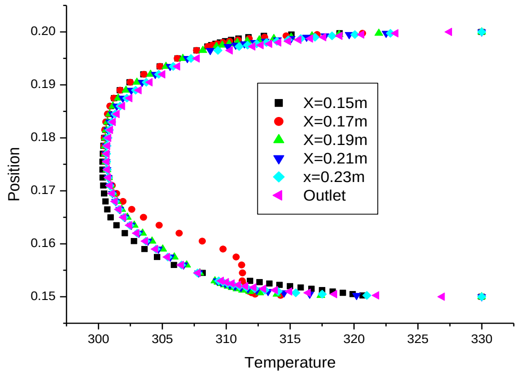

Fig 11. Temperature distribution at different X(C)

Fig 12 and 14 represent the length wise distribution of top-wall based Nusselt number of the lower horizontal limb and upper horizontal limb respectively

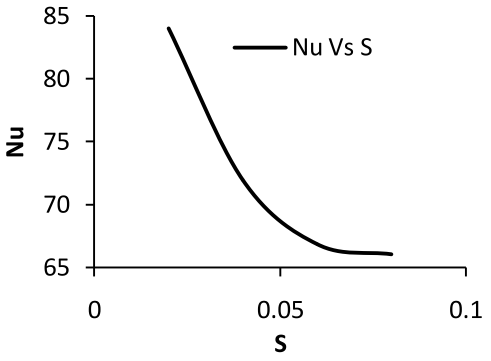

Fig 12. Nu at different position X(A)

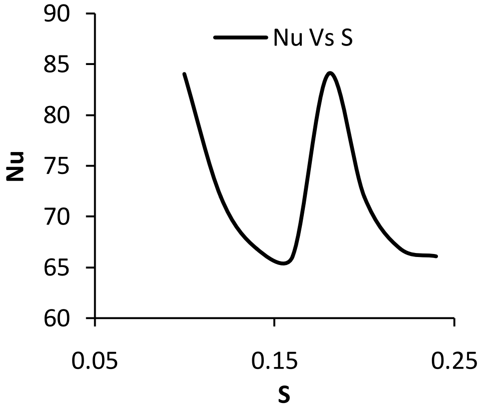

Fig 13. Nu at different Position Y(B)

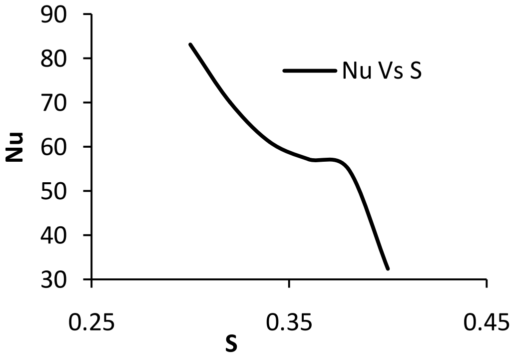

Fig 14. Nu at different Position X(C)

Fig 13 illustrates Nusselt number distribution along the left vertical wall of the vertical limb of the elbow duct. In the Fig 12, Nusselt number is maximum at the entrance region and decreases along the length of the lower limb. Similar phenomenon is observed in the upper horizontal limb as shown in Fig 14. However, local Nusselt number gets some higher value at a smaller region as shown in the Fig 12 and then it decreases in the similar manner as earlier.

## 5. CONCLUSIONS

The detailed velocity distribution has shown the presence of strong re-circulations at different zones of the geometry considered. The recirculation bubbles strongly affect the main flow. Temperature remains almost constant at the recirculation zones. The augmentation of Nu mainly takes place in the entrance region and then decreases along the elbow length.

## 6. REFERENCES

1. Cheng, K.C., Guang, J. and Hwang, G. J., 1969, "Numerical Solution for Combined Free and Forced Laminar Convection in a Horizontal Rectangular Channel", J. Heat Transfer, 91, 59-66.

2. Patankar, S.V., and Spalding, D. B., 1972, "A Calculation Procedure for Heat, Mass and Momentum Transfer in Three Dimensional Parabolic Flows", Int. J. Heat and Mass Transfer, 15, 1787-1806.

3. Cheng, C. H. and Weng, C. J., 1993, "Developing Flow of Mixed Convection in a Vertical Rectangular Duct with One Heating Wall", Numerical Heat Transfer, Part A, 240, 479-493.

4. Payvar, P., 1997, “Heat Transfer Enhancement in Laminar Flow of Viscoelastic Fluids through Rectangular Ducts”, Int. J. Heat Mass Transfer, 40(3), 745-756.

5. Yang, W. and Kuan, B., 2006, “Experimental Investigation of Dilute Turbulent Particulate Flow inside a curved 90-degree bend”, Chemical Engineering Science, 61, 3593-3601.

6. Spedding, P.L. and Benard, E., 2007,”Gas-liquid two phase flow through a vertical 90-degree elbow bend”, Experimental Thermal Fluid Science, 31, 761-769.

7. Mandal, A., Bhattacharjee, S., Debnath, R., Roy, D. and Majumder, S., 2010, “Experimental Investigation of Turbulent Fluid Flow through a Rectangular Elbow”, Int. J. Engg. Science and Technology, 2(6), 1500-1506.

8. Debnath, R., Bhattacharjee, S., Mandal, A., Roy, D. and Majumder, S., 2010, “A Comparative Study with flow Visualization of Turbulent fluid flow in an Elbow”, Int. J. Engg. Science and Technology, 2(9), 4108-4121

9. Özisik, M. N., 1985, “Heat Transfe: A Basic Approach”, McGraw-Hill Book Company, Singapore, Int. Edition, 310-317.

10. Rokni, M. and Gatski, T.B., 2001, “Predicting Turbulent Convective Heat transfer in fully developed duct flows”, Int. J. Heat and Fluid Flow, 22, 381-392.

11. Majumder, S. and Sanyal, D., 2010, “Relaminarization of Axisymmetric Turbulent Flow with Combined Axial Jet and Side Injection in a Pipe” ASME Journal of Fluids Engineering, 132:101101-1-101101-6.

12. Jang, J., Chiu, H. and Yan, W., 2010, “Wall Transpiration Effects on Developing Mixed Convection heat transfer in Inclined Rectangular Ducts’, J. Marine Science and Technology, 18(2), 249-258.

13. Launder, B. E. and Spalding, D.B., 1974, “The Numerical Computation of Turbulent Flows”, Computer Methods in Applied Mechanics, 3, 269-289.

## 7. NOMENCLATURE

| Symbol | Meaning | Unit |
|---|---|---|
| $C_{s1}$ | Empirical Constant | - |
| $C_{s2}$ | Empirical Constant | - |
| $C_{\mu}$ | Empirical Constant | - |
| $u$ | Average velocity along X axis. | (m/s) |
| v | Average velocity along Y Axis. | (m/s) |
| $U_{in}$ | Average inlet velocity | (m/s) |
| k | Turbulent Kinetic energy | $(m^2/S^2)$ |
| $L_1$ | Length of the bottom limb | (m) |
| $L_2$ | Depth of the duct | (m) |
| $L_3$ | Length of the upper limb | (m) |
| $Re$ | Reynolds number | - |
| μ | Dynamic viscosity | Kg/(m.s) |
| $\mu_l$ | Molecular or laminar viscosity | $(N\text{-}s/m^2)$ |
| $\mu_t$ | Turbulent viscosity | $(N\text{-}s/m^2)$ |
| $\mu_{eff}$ | Effective viscosity | $(N\text{-}s/m^2)$ |
| ε | Turbulence kinetic energy dissipation rate | $(m^2/s^3)$ |
| $\sigma_k$ | Prandtl number of the turbulent kinetic energy | - |
| $\sigma_\varepsilon$ | Dissipation Energy | - |
| $\rho$ | Density | $(Kg/m^3)$ |
| s | Station position | (m) |

## 8. MAILING ADDRESS

**Dr. Snehamoy Majumder**
Associate Professor
Department of Mechanical Engineering
Jadavpur University, Kolkata-700032,
WEST BENGAL, INDIA
**Phone:** +919831369134
**E-mail:** srg_maj@yahoo.com